Activity #2: Math

Title: An Investigation of the Sinusoidal Curve (Teacher version)

Note to students: Lab teams of three or four students are required for this activity.

National Math Standards addressed:
Content Standards:
Algebra Expectations: Students will understand and perform transformations such as combining, composing, and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions; students will use symbolic algebra to represent and explain mathematical relationships; students will judge the meaning, utility and reasonableness of the results of symbolic manipulations, including those carried out by technology; students will identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; students will draw reasonable conclusions about the situation being modeled.
Measurement Expectation: Students will make decisions about units and scales that are appropriate for problem situations involving measurement.
Process Standards:
Problem Solving Expectation: Students will reflect on the process of mathematical problem solving.
Communication Expectation: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
Connection Expectation: Students will recognize and apply mathematics in contexts outside of mathematics.
Representation Expectation: Students will use representations to model and interpret physical, social, and mathematical phenomena.

***Purpose*:**
- **To understand the definition of sinusoidal curve**
- **To recognize a possible sinusoidal curve**
- **To express a curve in sinusoidal form**
- **To generate, using technology, a sine curve and find its mathematical representation**
- **To recognize phenomena that might generate these curves and discuss advantages of the sine representation**

This activity has been modified from Texas Instruments CBL System Experiment Workbook ©1994, Experiment M3: Models of Oscillating Objects.

***Materials*: TI graphing calculator, TI-CBL and motion detector (Vernier probe) or TI-CBR, HOOK program for TI graphing calculator, TI-Graph Link, spring, clamp, cardboard or index card, tape, mass, meter stick or yardstick or similar piece of wood, TI overhead grapher and view screen (optional)**

Procedure:

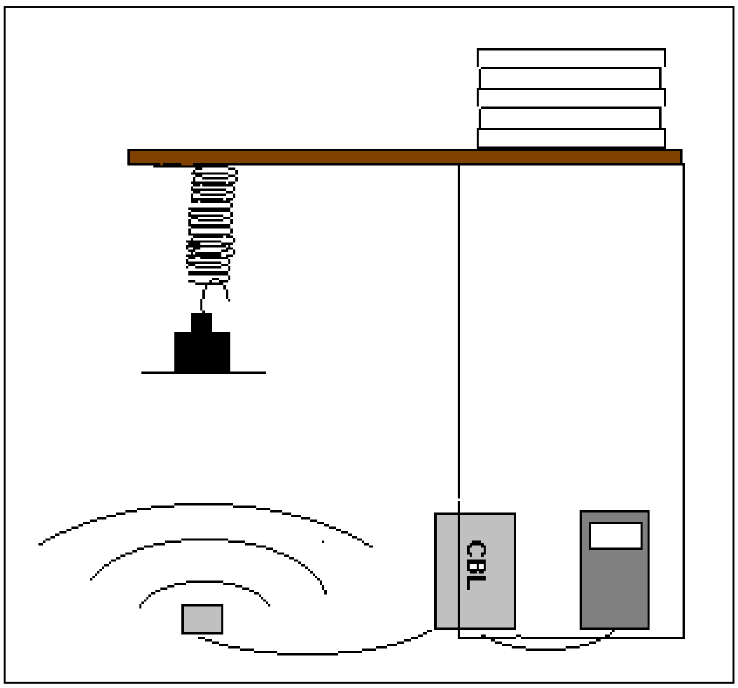
A. Equipment Setup as follows (see diagram):

1. Attach the spring to the end of the meter stick.

2. Place the meter stick on a table or file cabinet and clamp tightly or hold securely in place with a stack of math books. Make the overhang as short as possible to lessen the effect of flex in the stick.

3. Connect the CBL unit to the TI calculator and connect the motion detector to the I/O port. Be sure to press the cable ends in tightly.

4. Place the motion detector on the floor directly under the spring. Be sure the spring as the mass moves up and down gets no closer than 1.5 feet to the motion detector.



B. Experiment Steps as follows:

1. Turn on CBL and start the HOOK program. Data collection starts when you press TRIGGER on the CBL. When the data is collected and sent to the TI calculator, the distance in feet is stored in L_2 and the time in seconds is store in L_1, the data points will be displayed on the calculator.

2. Attach the mass to the end of the spring. Attach a piece of cardboard or an index card to your mass to serve as a target for the motion detector.

3. Pull the mass straight down and release so that oscillation is up and down only. Press TRIGGER on the CBL. (Try to pick a spring and mass that will generate an oscillation with an almost unnoticeable dampening so that the sine curve generated will be easier to formulate.)

4. Using the TI Graphlink, print the displayed data points screen of your TI. (A good set of data points will show at least three highs or lows.)

5. On this printout, label the high points, the maximum points, and label the low points, the minimum points.

C. Analysis as follows:

$$y = a \sin (bx - c) + d$$

1. The value of a represents the amplitude and opening direction of the curve. Write your value for a, correct to three decimal places. ______________
For best results, store this value in your calculator. What variable did you select to store a? ____

2. The value for b affects the period of the graph. Remember that b is a positive real number. What period did you find, correct to three decimal places? ______________ Remember, the period is the length of one complete cycle and is given by, period = $\frac{2\pi}{b}$**.**
For best results, store this value in your calculator. What variable did you select to store b? ____

3. The value of c affects the phase shift. Remember, the phase shift is given by $\frac{c}{b}$**. If c is negative, the phase shift is left and if c is positive, the phase shift is right. Determine the value for c. What was your value for c, correct to three deciamal places? __________ For best results, store this value in your calculator. What variable did you select to store c? ____**

4. Lastly, you need to calculate the value for d. Remember that d describes the vertical shift in the graph. What value did you calculate for d, correct to three decimal places? __________

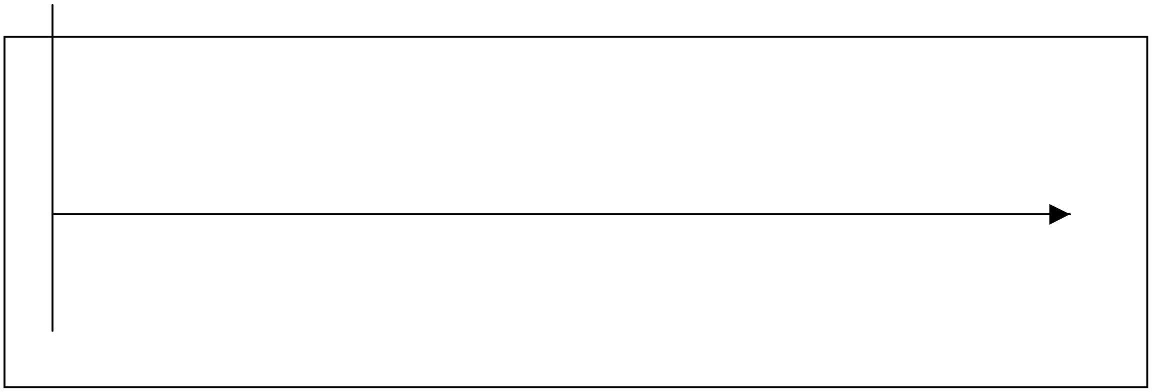
5. You are now ready to compare the data points graphed to the sinusoidal representation you have just found. Using a second TI graphing calculator, substitute the exact values for a, b, c, and d in the sinusoidal model, $y_1 = a \sin (bx - c) + d$**. Enter this as a function in the second TI calculator, and compare the two graphs (TRACE). Write your equation here.**

____

Do you think your sinusoidal form is a good model for the data you collected in this experiment?________ Why or why not?

____
____
____
____
____

D. Extension:
Sketch the graph that models the path of the mass over a long period of time.



Reflect on this experiment and on the accuracy of the math model you discovered. Describe what conditions, if any, which could occur that might make the model less than perfect. Explain how you might refine your model to reflect the changes caused by these conditions.

Following this experiment, a discussion of the dampening factor should occur. Asking whether the action of the spring will indefinitely be modeled correctly by the sinusoid will generate many comments from students. How will the sine curve as it represents the oscillation of the spring be affected over time? Do you think you could make any adjustments to the mathematical model to correct for this dampening? If so, what would you change in the model --- a, b, c and/or d? Would these changes be constant? Challenge students to find an equation of a sine whose graph appears to display dampening. Have students show their graphs using the TI overhead grapher and view screen. Look at $f(t) = a(t) \sin (bt - c) + d$. Have students graph the following, if they are having difficulty.
1. $y = x \sin x$, 2. $y = (.8^x) \sin 10x$
Before ending, discuss $y = a \cos (bt - c) + d$ and decide what it represents.

Activity #1 and #2 link to, Activity 1 from the science component, Title: An Investigation into Transverse Waves.

The following web sites and articles provide enrichment and support for this activity:
1. http://www.falstad.com/mathphysics.html
2. http://www.coolmath.com/dampfunction1.htm
3. **Sine Curve orbits, "A Room with a View",** http://www.NASAexplores.com
4. http://www.mathpages.com/home/kmath210/kmath210.html